(1) Write a general equation for the following sequences: (For example, a general equation for the sequence 2, 5, 10, 17, 26, ··· would be $a_n = n^2 + 1$. The equations should be closed form, and they should be single equations. For example “$a_n = \frac{n}{2}$ if $n$ is even, and $a_n = \frac{n+1}{2}$ if $n$ is odd.” wouldn’t be an acceptable answer for part c.)

(a) 3, 5, 7, 9, 11, ···

(b) 1, 5, 7, 17, 31, 65, 127, 257, ··· (Hint, remember that $(-1)^i = 1$ for even $i$, and $(-1)^i = -1$ for odd $i$.)

(c) 1, 1, 2, 2, 3, 3, 4, 4, ··· (BONUS points if you can do this without using any “rounding” operators such as floor or ceiling.)

(2) Write a closed form formula for the following summations: (For example, a closed form formula for $\sum_{i=1}^{n} 2i$ would be $n(n + 1)$.)

(a) $\sum_{i=1}^{n} (2i + 3 + 9^i)$

(b) $\sum_{i=1}^{n} ((-1)^i \cdot 9^i)$

(c) $\sum_{i=1}^{n} \sum_{j=1}^{i} ij$

(d) $\sum_{i=1}^{n} \sum_{j=1}^{i} j$ (Hint, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$. Also note that the interior summation is from $j = 1$ to $i$, not $n$.)

(3) Give a recursive definition for the following sequence: (Be sure to specify the base case(s).) 1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, ···

(4) To be a member of the Sons of the American Revolution, you have to have “at least one ancestor who supported the cause of American Independence during the years 1774-1783”. Let’s suppose that everyone who fits this criterion is a member.
If we define $S$ to be the set of people who “supported the cause of American Independence during the years 1774-1783”, and we define $p(x, y)$ to be “$x$ is a parent of $y$”, then give a recursive definition of $R$: the set of members of the Sons of the American Revolution in terms of $S$ and $p$.

(5) BONUS: The Fibonacci numbers are defined by the equation

$$f_n = f_{n-1} + f_{n-2}$$

where $f_1 = f_2 = 1$.

Prove that, in the limit as $n \to \infty$, $\frac{f_{n+1}}{f_n} = \phi$, where $\phi$ is the Golden Ratio, given by $\frac{1 + \sqrt{5}}{2}$.

Hint, you may want to prove that the following closed form equation gives $f_n$:

$$f_n = \frac{(1 + \sqrt{5})^n - (1 - \sqrt{5})^n}{2^n \sqrt{5}}$$

then use this as a lemma in your proof.